kTun

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Abstract

kTun is a computational model developed to compute the impact of a high-speed train travelling through a tunnel. This paper outlines the modelling methodology and documents the verification and validation of the algorithms applied. This paper is not intended as an academic paper but rather to be a more complete document outlining the development process.

1 Introduction

1.1 General

A train entering a tunnel generates a compression wave at the entry portal. This wave propagates along the tuunel at the speed of sound in front of the train. The train movement will displace air which will generate friction at the tunnel and train walls. These effects combine to produce a pressure gradient which results in a pressure rise in front of the train. On reaching the exit portal of the tunnel, the compression wave is partially reflected back as an expansion wave. The part that is not reflected exits the tunnel portal as a micropressure wave (see e.g. [1] and references therein). This micropressure wave could cause a sonic boom that may lead to structural vibration and noise pollution in the surrounding environment. The entry of the tail of the train into the tunnel produces an expansion wave that moves through the annulus between the train and the tunnel. When the expansion pressure wave reaches the entry portal, it is reflected towards the interior of the tunnel as a compression wave. These compression and expansion waves propagate backwards and forwards along the tunnel and experience further reflections when meeting with the nose and tail of the train or reaching the entry and exit portals of the tunnel until they eventually dissipate completely [2]. The presence of this system of pressure waves in a tunnel affects the design and operation of trains, and they are a source of energy losses, noise, vibrations and aural discomfort for passengers. These problems are even worse when two or more trains are in a tunnel at the same time.

1.2 Purpose of kTun

There are already many mathematical models that exist that allow the simulation of trains in tunnels. These cover both compressible and compressible flow, implicit and explicit train movement, full network models and are industry standards. These models use a range of different numerical methods to simulate the underlying physics. the most common is the method of characteristics however finite volume and discontinuous galurkin methods have also been used.

kTun uses the variable area one-dimensional form of the Euler equations and solves them using the finite volume method. The formulation allows for unsteady, compressible flow and solves for continuity of mass, momentum and energy.

kTun has been developed to allow simple trade offs to be made and to allow engineering judgement to be exercised. As additional functionality is required this can be included provided the inputs can be resolved to the required mass, momentum and energy terms.

1.3 Layout of document

Section 2 describes the mathematics behind the model approach.

2 Mathematical Basis

2.1 Nomenclature

Symbol	Meaning	Units
ρ	Density	kg/m^3
P	Pressure	Pa
u	Velocity	m/s
A	Area (cross sectional)	m^2
T	Temperature	K
t	Time	s
x	Spatial coordinate	m

Table 1: Nomenclature.

2.2 Basic Equations

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho A\\ \rho u A\\ \rho E A\end{pmatrix}+\frac{\partial}{\partial x}\begin{pmatrix}\rho u A\\ \rho u^2 A+PA\\ \rho u H A\end{pmatrix}=\begin{pmatrix}0\\ \alpha\\ \beta\end{pmatrix} \tag{1}$$

Where α and β are source terms which will be discussed in slightly later on in this section. Also note that

$$E = e + \frac{1}{2}u^2 \tag{2}$$

$$H = E + \frac{P}{\rho} \tag{3}$$

$$e = c_v T \tag{4}$$

The analysis assumes that the working fluid will be an ideal gas and thus equation 5 applies.

$$P = \rho R T \tag{5}$$

The Finite Volume Method (FVM) is extremely common in computational fluid dynamics. It is a versatile discretization technique for partial differential equations that arise from physical conservation laws. The approach breaks the domain up into a number of control volumes which for the rest of this paper will be abbreviated as CV(s). The CVs can vary in length and cross-sectional area. At each CV boundary a flux exchange is calculated. This flux exchange ensures continuity at all points within the domain.

The implementation of the FVM method in kTun is first order accurate in space and does not limit numeric oscillations. This can cause some volatility in the specifics where sharp changes in state (typically pressure in this application) are predicted. Numerical limiting is possible and may be reviewed as a future development.

The FVM works with conserved and primitive variables. The conserved variables are converted into primitive variables. The primitive variables are used to determine the flux exchanges. This is then used, in combination with the source terms to updated the conserved variables. This process is then repeated at every time step in the analysis.

The conserved variables in this formulation are the terms in the time derivative of equation 1. The primitive variables are the pressure, velocity and density. The area variations are determined as a function of the tunnel geometry and train movement and are therefore fully known throughout each time step.

All variables are determined as the CV centered average value. However, all flux exchanges occur at the interface between CVs. It is therefore required to determine what the primitive variables are on the CV interface. This is a Riemann problem, a specific type of initial value problem. This is solved using a Riemann solver which takes the state, primitive variables in the cells on either

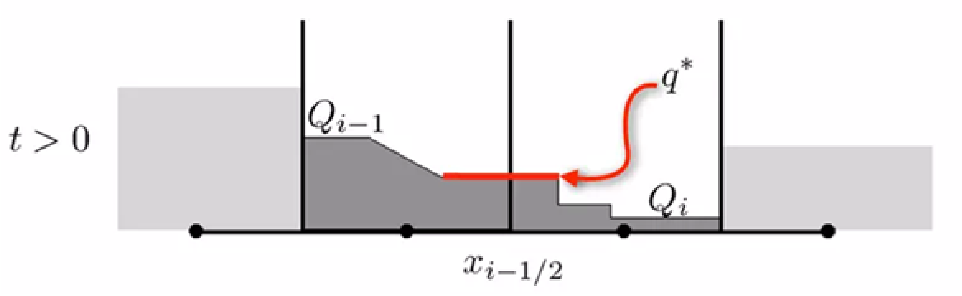

Figure 1: Cell interface and flux calculation.

side of the interface, and determines what the cell boundary would be. This is illustrated in figure 1.

The conserved quantities in each cell are used to determine the flux at the cell interface. This is achieved using a four step process. This is repeated every timestep. Steps 1-3 are essentially resolving the spatial derivative whilst step 4 is resolving the temporal derivative. In summary the four steps are:

1. Convert conserved variables into primitive variables
2. Solve the Riemann problem between all neighbouring cells from the primitive variables
3. Calculate the fluxes from the primitive variables
4. Update the conserved variables based upon the fluxes from the neighbouring cells

The execution of each of these four steps are outlined in turn

2.2.1 Conversion of conserved variables to primitive variables

The conserved quantities in kTUN and their units are:

- Mass (kg/m)
- Momentum (kg/s)
- Energy (kgm/s^2)

These are converted into the primitive variables of density, velocity and pressure using equations 6, 7 and 8. A is the average area across the cell.

$$\rho = \frac{m}{A} \tag{6}$$

$$u = \frac{q}{m} \tag{7}$$

$$P = (\gamma - 1)(\frac{E}{A} - \frac{1}{2}\rho u^2) \tag{8}$$

This conversion to primitive variables is undertaken for all cells across the domain simultaneously before moving to step 2. The area used to convert mass and Energy will derived from the cell volume and length at the start of the timestep.

2.2.2 Solve the Riemann Problem

In kTun the convention is that left side of the cell is lower in x (chainage) than the right side of the cell. The Riemann solver is fed the primitive variables from the left and right sides of an interface and then samples the solution to identify the values of the primitive variables at the interface between the cells. See Appendix A for notes on the Riemann problem and verification of the kTUN Riemann solver. The Riemann problem solution determines the properties of pressure, density and velocity at the interface between the two cells.

2.2.3 Calculate the fluxes

The sampled values of the primitive variables are used to determine the flux that occurs at the cell interface. This is calculated using equations 9, 10 and 11. In these equations A_i refers to the cross-sectional area at the cell interface.

$$F_m = \rho u A_i \tag{9}$$

$$F_q = A_i(\rho u^2 + P) \tag{10}$$

$$F_E = (\frac{\gamma}{(\gamma - 1)}P + \frac{1}{2}\rho u^2)uA_i \tag{11}$$

2.2.4 Update the conservered variables

Once the fluxes have been calculated the conserved variables can be updated. These are updated by applying the fluxes calculated in step 3 to the conserved quatities by adding flux from the left and subtracting flux to the right. If there is a source term (α or β in the governing equations) to be included at a specific cell then this would be accounted for here by increasing or decreasing the appropriate conserved variable as part of this step. These steps can now be looped for as many timesteps as required to complete the simulation time required.

2.3 Timestep

The timestep used in kTUN applies the convergence condition developed by Courant-Friedrichs-Lewy. This uses the minimum value from all cells applying a CFL constant. This condition is shown in equation 12.

$$\Delta t = C_{cfl}\ min(\Delta x/\sqrt{\gamma\frac{P}{\rho} + u^2}) \tag{12}$$

Smaller coefficients will result in smaller timesteps. The CFL coefficient is a user specifiable value and selection may depend upon the application. In all validation and verification analysis in this report the CFL coefficient used is quoted.

3 Testing FVM formulation

3.1 Sod Shock Test

A common test case for compressible flow solvers is the Sod shock problem. This test dates from 1978 and consists of a one dimensional Riemann problem. A tube of unit lengthh holds gas either side of a diaphragm which is located half way along the tube. The gases are at different pressure and densities as defined in equation 13.

$$\begin{pmatrix}\rho_L \\ u_L \\ P_L\end{pmatrix} = \begin{pmatrix}1 \\ 0 \\ 1\end{pmatrix}, \begin{pmatrix}\rho_R \\ u_R \\ P_R\end{pmatrix} = \begin{pmatrix}0.125 \\ 0 \\ 0.1\end{pmatrix} \tag{13}$$

The diaphragm bursts at t=0 and the time evolution of the problem can be solved analytically. Therefore, the computer code can be tested against this to verify that the underlying method is working correctly.

It is common to test the solution validity after 0.2s. At this time the compressible flow has developed sufficiently to show all phase of the solutions waves but has not run for sufficient time to cause an interaction at the end of the tube. The results of this simulation can be seen in figure 2. kTUN was set up with 100 cells and a CFL coefficient of 0.9.

kTUN standard graphical output consists of six panes. These show the area and Mach number on the top row, the density and velocity in the middle row and the pressure and temperature in the bottom row. The analytical result in all cases is shown as a black line while the predictions of kTUN are shown as coloured lines. In all charts the x axis shows chainage along the model.

The SOD shock case is a dimensionless problem but the standard SI unit labels have been left in the chart so that recoding this routine was not required. The area chart is trivial and solely confirms the Sod shock tube is of unit area cross section. The Mach number also has not got

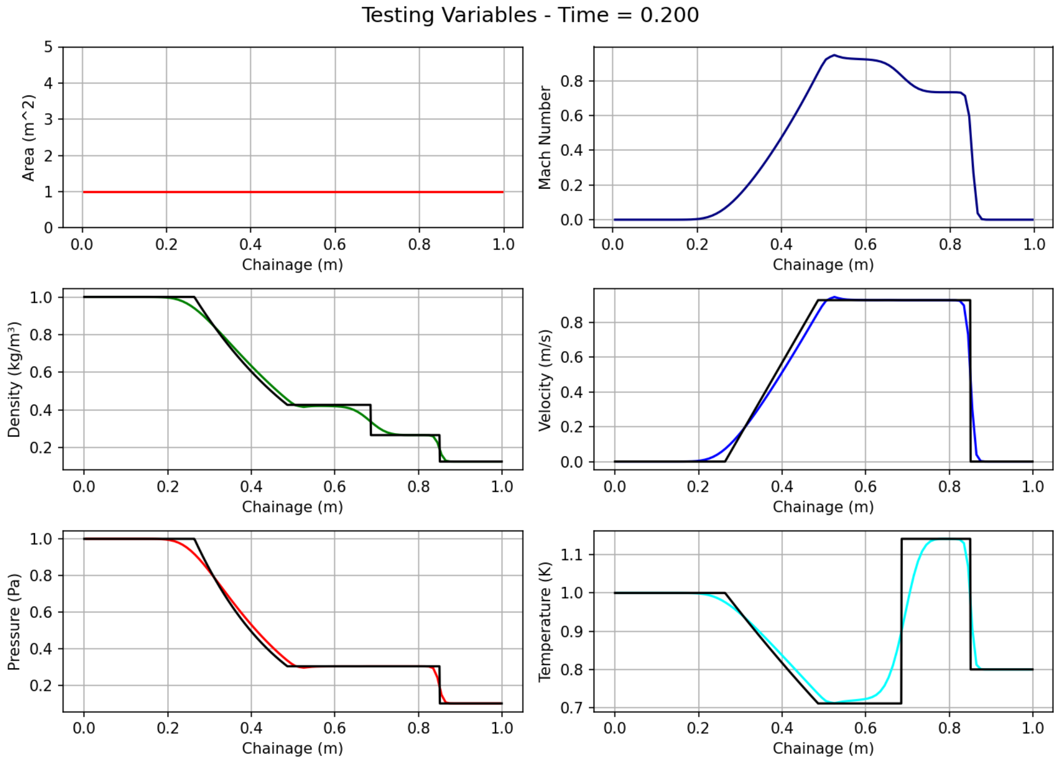

Figure 2: Verification of Sod shock analysis

an analytical solution associated with it. This provided solely for information. The agreement between the Density, velocity, pressure and temperature traces is strong in all areas except those associated with the straight shock discontinuity. This region is often where numeric solutions require adaption or limiters to be applied to improve the solution accuracy. The resolution can also be improved by increasing the number of cells either in this region alone or across the whole domain.

The Sod shock case is undertaken on a constant cross-sectional area tube. Therefore, this test has not considered if the variable area formulation is correct. However, the strong agreement between the underlying code and the analytical solution demonstrates that the exchange between the cells is working correctly.

3.2 Train

The train is modelled as an area change in the tunnel. The train is moved through the tunnel based on the current train speed and rate of acceleration. This determines where the train will be at the beginning and end of the time step. At the time of moving the train all area and spatial derivatives associated with the train are also calculated for use in the rest of the analysis.

The source terms α and β that were mentioned in the equation 1 are a result of changes in friction and train movement. There is also a dependency on any changes in tunnel cross-sectional area that may be caused by expansions or contractions locally. the source term α is defined in equation 14 and β us defined in equation 15

$$\alpha = \alpha_{va} + \alpha_{fr} + \alpha_{cr} \tag{14}$$

$$\beta = \beta_{va} + \beta_{fr} + \beta_{cr} \tag{15}$$

References

[1] A. E. Vardy, "Generation and alleviation of sonic booms from rail tunnels," *Proceedings of the Institution of Civil Engineers, Engineering and Computational Mechanics*, vol. 161, no. EM3, pp. 107–119, 2008.

[2] R. G. Gawthorpe and C. Pope, "Aerodynamics and ventilation of vehicle tunnels. the measurement and interpretation of transient pressures generated by trains in tunnels," *Second International Symposium on the Aerodynamics and Ventilation of Vehicle Tunnels*, pp. 35–54, 1976.

4 Appendix A - Riemann Problem

To be written